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Generation of a class of SU(1, 1) coherent states of the Gilmore–Perelomov type and a class of SU(2) coherent states and their superposition

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Abstract
In this paper, we first suggest a scheme for the generation of a particular class of Gilmore–Perelomov-type SU(1, 1) coherent states, which may be established as nonlinear coherent states. The proposal employs a two-level atom that interacts with a single-mode quantized cavity field (by using an intensity-dependent Jaynes–Cummings model) and at the same time a strong external classical field. The time evolution of the system first leads to the generation of a superposition of SU(1, 1) coherent states. Depending on the initial states of the atom and the field which may be appropriately prepared, and also under the conditions in which the atom is detected (in the excited or ground state) after the occurrence of the interaction, the field will be collapsed to arbitrary combinations or a single class of Gilmore–Perelomov-type SU(1, 1) coherent states. Then, it is shown that, following a similar procedure, our proposed scheme can successfully generate various superpositions and, in particular, a single class of SU(2) coherent states, too.

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1. Introduction
Glauber, Sudarshan and Klauder employed standard coherent states to effectively explain the properties of radiation fields and laser light [1–3]. These states are constructed by the eigenvalue equation \( a|\alpha\rangle = \alpha |\alpha\rangle \) or via the action of the unitary displacement operator \( D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \) on the vacuum state \( (a \text{ and } a^\dagger \text{ are, respectively, bosonic annihilation and creation operators and } \alpha \text{ is a complex number, in such way that } \langle \alpha|a^\dagger a|\alpha\rangle = |\alpha|^2 \text{ is the mean number of photons}). \) On the other hand, nonlinear coherent states (NLCSs),

\[
|\alpha, f\rangle = N^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!} [f(n)]!} |n\rangle, \quad \alpha \in \mathbb{C},
\]

are an important class of generalized coherent state, which satisfy the eigenvalue equation \( A|\alpha, f\rangle = \alpha |\alpha, f\rangle \), where \( A = af(n) (A^\dagger = f^\dagger(n) a^\dagger) \) is the deformed annihilation (creation) operator and \( f(n) \) is an operator-valued function of the intensity of light with \( n = a^\dagger a \). These states were introduced by Matos and Vogel [4] and Man’ko et al [5]. It is shown that such states can appear as a hypothetical frequency blue shift in high-intensity photon beams [6]. As a further extension, NLCSs on the circle have been deduced in [7]. In addition to the ability of NLCSs to describe the motion of the trapped ion [8, 9], these states have been used to illustrate the behavior of a quantum harmonic oscillator which is confined in a one-dimensional infinite well [10, 11]. It has been shown that photon-added coherent states that were introduced by Agrawal and Tara [12] and then generated experimentally by Zavatta et al [13] may be known as NLCSs with a particular nonlinearity function [14]. The \( f \)-coherent bound state that has been constructed for the Morse potential was established in [15]. Also, binomial states [16, 17] and negative binomial states [18] can be viewed as NLCSs.
Nonclassical properties of NLCSs and their superpositions, such as squeezing, sub-Poissonian statistics, the antibunching effect and their phase properties, have been studied in the literature [19–22]. Due to their nonclassical features and hence their various applications, the generation of NLCSs is a very important issue in quantum optics. The generation of NLCSs in a lossless micromaser cavity, under intensity-dependent Jaynes–Cummings interaction, has been investigated in [23]. It is shown there that even and odd NLCSs (as special kinds of superposed states) can be produced in a two-photon micromaser. Another scheme for the generation of even and odd NLCSs has been presented in [24]. A proposal for the generation of motional NLCSs and their superpositions has recently been proposed in [25]. A theoretical scheme for the generation of Gazeau–Klauder coherent states [26, 27] through intensity-dependent degenerate Raman interaction recently was proposed by one of us in [28].

In this paper, first we want to suggest a new scheme for the generation of a particular class of NLCSs and their superpositions. To achieve this purpose, we consider a Hamiltonian that describes the interaction of a two-level atom with a single-mode cavity field via an intensity-dependent coupling version of the Janes–Cummings model, together with a strong external classical field. A similar Hamiltonian but with constant coupling has recently been used for the production of superpositions of squeezed coherent states [29, 30, 31]. However, in the type of the interaction which we will introduce in this paper, one cannot generally define a unitary displacement operator by means of $A$ and $A^\dagger$, since the commutation of $A$ and $A^\dagger$ is again an operator. To overcome the problem, we employed a particular form of $f(n)$ corresponding to the trigonometric potential, which has been studied in [32, 33]. It is illustrated that by considering such a nonlinearity function, $A$, $A^\dagger$ and their commutator satisfy the $su(1,1)$ Lie algebra. Therefore, construction of the displacement operator with the help of $A$ and $A^\dagger$ becomes possible, where the action of this operator on the vacuum state of field produces $SU(1,1)$ coherent states of the Gilmore–Perelomov type [34, 35]. It was previously proved that these states are also NLCSs with a particular nonlinearity function [36]. Fortunately, our presented formalism can be applied to a class of finite-dimensional states, too. Towards this goal, it is illustrated in [32] that if one considers the nonlinearity function of the modified Pöschl–Teller potential, then $A$, $A^\dagger$ and their commutator satisfy $su(2)$ Lie algebra. Accordingly, the action of the corresponding displacement operator on the vacuum state of field generates $SU(2)$ coherent states.

This paper is organized as follows. In the next section, the modified potential, its nonlinearity function and the construction of corresponding NLCSs with the associated displacement operator are presented. In section 3, by considering an intensity-dependent Hamiltonian, the time evolution of the system is studied and the generation of $SU(1,1)$ coherent states (which are also of NLCSs structure) in addition to their combinations are illustrated. The $SU(2)$ coherent states associated with a particular physical system are generated in section 4. Finally, a summary and the conclusions are presented in section 5.

2. The deformed displacement operator

One of the ways of constructing coherent states is by the action of a unitary operator on the vacuum of the field, i.e. they can be defined as orbits of a unitary operator acting on the vacuum of the field. On the other hand, with the help of the operators $A$, $A^\dagger$ together with the auxiliary operators $\tilde{B} = a_{\lambda} f_{\lambda}$ and $\tilde{B}^\dagger = a_{\lambda}^\dagger f_{\lambda}$, two kinds of (nonunitary) displacement-type operators can be obtained as $D_1(\alpha) = \exp(\alpha B^\dagger - \alpha^* A)$ and $D_2(\alpha) = \exp(\alpha A^\dagger - \alpha^* B)$ with the following actions on the vacuum [22, 36, 37]:

$$
|\alpha, f_1\rangle = D_1(\alpha) |0\rangle = N_1 (|\alpha|^2)^{-1/2} \sum_{n=0}^\infty \frac{\alpha^n}{\sqrt{n!}} |f(n)| |n\rangle,
$$

$$
|\alpha, f_2\rangle = D_2(\alpha) |0\rangle = N_2 (|\alpha|^2)^{-1/2} \sum_{n=0}^\infty \frac{\alpha^n |f(n)|}{\sqrt{n!}} |n\rangle,
$$

yielding two distinct sets of NLCSs. In (2) and (3), $N_1$ and $N_2$ are, respectively, appropriate normalization factors that can be determined. The two resultant classes of NLCSs have been called the dual pair [36, 37]. An advantage of this approach is that such displacement-type operators can be established for any class of NLCSs. Nevertheless, it is illustrated in [32] that the construction of a unitary displacement operator with $A$ and $A^\dagger$ is also possible if one considers the particular nonlinearity function associated with the (modified) trigonometric potential $V(\nu) = U_0 \tan^{-1}(\nu x)$, where $U_0$ is the strength of the potential and $b$ is its range. Using the approach of Man’ko et al [5], the nonlinearity function that corresponds to this potential is given by

$$
f_1(n) = \frac{\hbar^2}{2\mu \Omega} (n + 2\lambda - 1),
$$

where $\Omega$ is the frequency of the field, $\mu$ is the mass of the particle, $\lambda$ is related to the potential strength and for the range one reads as $\lambda(\lambda + 1) = \frac{2\hbar^2}{\hbar^2+b^2}$ (the subscript $1$ in $f_1(n)$ and in what follows expresses the connection of the states or operators with $SU(1,1)$ coherent states). With the particular choice in (4), the commutation relation $[A_1, A_1^\dagger] = \frac{2\hbar^2}{\hbar^2} (\lambda + n)$ is obtained. Then, the redefinition of $K_1^- = 2\frac{\hbar^2}{2\mu \Omega} A_1$, $K_1^+ = 2\frac{\hbar^2}{2\mu \Omega} A_1^\dagger$ and $K_1^z = \lambda + n$ leads to the commutation relations $[K_1^+, K_1^-] = \pm K_1^z$ and $[K_1^+, K_1^z] = 2 K_1^z$, which are the well-known $su(1,1)$ Lie algebra. Now, following the Gilmore–Perelomov approach to the group theoretical construction of coherent states, one has [34, 35]

$$
D_{f_1}(\alpha) |0\rangle = \exp(\xi K_1^z - \xi^* K_1^-) |0\rangle = \exp(\xi K_1^z) (1 - |\xi|^2)^{K_1^z} \exp(-\xi^* K_1^-) |0\rangle
$$

$$
= (1 - |\xi|^2)^{K_1^z} \sum_{n=0}^{\infty} \frac{\Gamma(n+2\lambda)}{n!\Gamma(2\lambda)} \xi^n |n\rangle,
$$

where we have used $\xi = \sqrt{\frac{\hbar^2}{2\mu \Omega}} \alpha$ and $\xi = \frac{\hbar}{\mu \Omega} \tanh \xi$. Also, by redefining $\eta(\alpha) = \sqrt{\frac{\hbar^2}{2\mu \Omega}} \xi$, the resulting state in (5) can be
rewritten as
\[
|\eta_1(\alpha), f_1\rangle = \left(1 - \frac{\hbar^2}{2\mu\Omega} |\eta_1(\alpha)|^2\right)^{\lambda} \sum_{n=0}^{\infty} \frac{\eta_1(\alpha)^n |f_1(n)\rangle}{\sqrt{n!}} |n\rangle,
\]
where by definition \([f_1(n)] = f_1(n) f_1(n-1) \cdots f_1(1)\). The equivalent states (5) and (6) are the well-known SU(1, 1) coherent states of Gilmore–Perelomov type and NLCSs, respectively.

3. Generation of a class of Gilmore–Perelomov-type SU(1, 1) coherent states

To achieve the goal of the paper, we consider the Hamiltonian in which a two-level atom interacts with a single-mode (quantized) cavity field via an intensity-dependent coupling in addition to an external classical field. With the resonance condition and with the rotating-wave approximation, the Hamiltonian has the following form (assuming \(\hbar = 1\)):
\[
H = \lambda (\hat{\sigma}_- e^{i\omega t} + \hat{\sigma}_+ e^{-i\omega t}) + \Lambda (A_1^\dagger \hat{\sigma}_- + A_1 \hat{\sigma}_+),
\]
where \(\sigma_- = |g\rangle \langle e|\) and \(\sigma_+ = |e\rangle \langle g|\) are the atomic lowering and raising operators, respectively. Also, the parameters \(\lambda\) and \(\Lambda\) are, respectively, the coupling coefficients of the atom with classical and quantized cavity fields, \(A_1\) and \(A_1^\dagger\) are the \(f\)-deformed ladder operators and \(\varphi\) is the phase of classical field. The coupling of the atom with the classical field is assumed to be stronger than the coupling of the atom with the cavity field. Note that we consider an intensity-dependent interaction of a two-level atom with a single-mode cavity field. A deep insight into the Hamiltonian (7) shows that the constant coupling coefficient \(\Lambda\) that has been used in [29, 30] is replaced by the intensity-dependent coefficient \(\Lambda f_1(n)\). This procedure is a typical nonlinear or \(f\)-deformed Janes–Cummings model (for a recent example of the usefulness of this approach see [38]). It is demonstrated that, in the strong classical field regime \((\lambda \gg \Lambda)\), the time evolution operator of the system is defined by [30]
\[
U(t) = R^{T^\dagger} (t) U_{\text{eff}} T(0) R,
\]
where the operators \(R\), \(T\) and \(U_{\text{eff}}\) are given by
\[
R = \exp\left[\frac{\pi}{4} (\sigma_+ - \sigma_-)\right] \exp\left[i\frac{\varphi}{2} \sigma_z\right] = \frac{1}{\sqrt{2}} \left(I + \sigma_+ - \sigma_-\right) \exp\left(i\frac{\varphi}{2} \sigma_z\right),
\]
\[
T(t) = \exp(i\lambda \sigma_z t),
\]
\[
U_{\text{eff}} = \exp\left[-\frac{i\Delta t}{2} (A_1^\dagger e^{-i\omega t} + A_1 e^{i\omega t}) \sigma_z\right].
\]

Finally, \(H(t)\) and its corresponding time evolution operator will be obtained by reversed transformations. In short, with the help of equation (8), the time evolution of the system can be readily evaluated.

To go further, we assume that the cavity is initially prepared in the vacuum of the field and the atoms are in a superposition of excited and ground states with equal weights, so that the atom–field state is then denoted by
\[
|\psi(0)\rangle = \left(|e\rangle + |g\rangle\right) |0\rangle.
\]
Using (8), the dynamics of the system will be obtained first by the action of \(R\) on the initial state of the atom–field system, i.e.
\[
R |\psi(0)\rangle = \left(\cos \frac{\varphi}{2} |e\rangle - i \sin \frac{\varphi}{2} |g\rangle\right) |0\rangle.
\]
Meanwhile, the operator \(U_{\text{eff}}\) will be treated as a displacement operator of the type which is introduced in (5), i.e.
\[
U_{\text{eff}} = U_{\text{eff}}\left(|e\rangle \langle e| + |g\rangle \langle g|\right) = e^{-i\Delta t (A_1^\dagger e^{-i\omega t} + A_1 e^{i\omega t})} |e\rangle \langle e| + i e^{i\omega t} (|e\rangle \langle e| + |g\rangle \langle g|).
\]

Note that, in obtaining (14), we have used the atomic unity operator \(I_\sigma\) and the fact that \(\sigma_+ |e\rangle = |e\rangle\) and \(\sigma_+ |g\rangle = -|g\rangle\). Consequently, the output of the action of \(U_{\text{eff}}\) on the state (13) is of the form
\[
\cos \frac{\varphi}{2} |\eta_1(\alpha), f_1\rangle |e\rangle - i \sin \frac{\varphi}{2} |\eta_1(\alpha), f_1\rangle |g\rangle.
\]

Finally, by acting \(R\) on the obtained state in (17), one obtains the final state of the atom–field system at time \(t\), given by
\[
|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-i\Delta t/2} e^{i\omega t} e^{-i\omega t} |\eta_1(\alpha), f_1\rangle + i e^{i\omega t/2} e^{i\omega t} |\eta_1(\alpha), f_1\rangle \right) |e\rangle
\]
\[
+ \frac{1}{\sqrt{2}} \left(-i e^{-i\omega t/2} e^{i\omega t} |\eta_1(\alpha), f_1\rangle \right) |g\rangle.
\]

such that \(|\Psi(t)\rangle\) is a general superposition of Gilmore–Perelomov-type SU(1, 1) coherent states. Now, depending on the situation in which the atom is detected, the state of the cavity field may be determined. Strictly speaking, if the atom is detected in an excited or ground state, the state...
of the field will be collapsed, respectively, to

$$\langle \Psi^\dagger \rangle_1 = \frac{N^+}{\sqrt{2}} \left( e^{-i\varphi/2} \cos \frac{\varphi}{2} |\eta_1(\alpha), f_1 \rangle + i e^{-i\varphi/2} \sin \frac{\varphi}{2} | -\eta_1(\alpha), f_1 \rangle \right)$$

(19)
or

$$\langle \Psi^- \rangle_1 = \frac{N^-}{\sqrt{2}} \left( e^{-i\varphi/2} \cos \frac{\varphi}{2} |\eta_1(\alpha), f_1 \rangle - i e^{-i\varphi/2} \sin \frac{\varphi}{2} | -\eta_1(\alpha), f_1 \rangle \right),$$

(20)

where $N^\pm = \sqrt{2}$ and we supposed that $\lambda t = 2k\pi$. This type of reduction of states is well known in the generation schemes of coherent state in the quantum optics framework (see, for instance, [39]). From equations (19) and (20) it is readily found that the cavity field has arrived at a combination of NLCSs which are also SU(1, 1) coherent states. It is worth mentioning that our present formalism demonstrates the interesting fact that $\eta_1(\alpha)$ in (6), as the parameter of the generated coherent states, can be interpreted by physical variables such as $\Omega$, $b$ and $\mu$.

Now note that the expansion coefficients in (19) and (20) are $\varphi$ dependent. This situation provides the possibility that by choosing an appropriate phase of the classical field, an arbitrary superposition of SU(1, 1) coherent states can be produced. For instance, in particular, by selecting $\varphi = 2\pi$ ($\varphi = \pi$), regardless of the atomic detection, the state of the field collapses to $| \eta_1(\alpha), f_1 \rangle$ ($| -\eta_1(\alpha), f_1 \rangle$), i.e. SU(1, 1) coherent states may be generated.

As another interesting situation, suppose that the atom is initially prepared in the excited state and the field is in the vacuum state, i.e.

$$| \psi(0) \rangle = | e \rangle | 0 \rangle. \quad (21)$$

Following a similar procedure, by acting the time evolution operator (8) on the initial state (21), the general state of atom–field at time $t$ can be produced as

$$\langle \Psi(t) \rangle_1 = \frac{1}{2} \left( e^{-i\varphi/2} |\eta_1(\alpha), f_1 \rangle + e^{i\varphi/2} | -\eta_1(\alpha), f_1 \rangle \right) | e \rangle + \frac{1}{2} \left( e^{i\varphi} e^{-i\varphi/2} |\eta_1(\alpha), f_1 \rangle - e^{i\varphi} e^{i\varphi/2} | -\eta_1(\alpha), f_1 \rangle \right) | g \rangle.$$ 

(22)

Choosing the phase of classical field such that $\varphi = 2k\pi$ at time $\lambda t = 2k\pi$, where $k \in \mathbb{Z}^+$, and detecting the atom in the excited (ground) state results in the generation of an even (odd) superposition of SU(1, 1) coherent states. These states are simply described by means of the relation

$$| \eta_1(\alpha), f_1 \rangle \pm = N^\pm | \eta_1(\alpha), f_1 \rangle \pm | -\eta_1(\alpha), f_1 \rangle \rangle,$$

(23)

where $N^\pm$ may be determined by the normalization condition as

$$N^\pm = \left( 2 \pm 2N^\dagger \sum_{n=0}^{\infty} \left( -|\eta_1(\alpha)\rangle \langle f_1(n) | \right)^2 \right)^{-1/2},$$

(24)

and $N^\dagger$ is the normalization factor of the original SU(1, 1) coherent states determined in (6). It is worth noting that the obtained even and odd superpositions of SU(1, 1) coherent states obey the following eigenvalue relation:

$$A^2 | \eta_1(\alpha), f_1 \rangle = \eta_1(\alpha)^2 | \eta_1(\alpha), f_1 \rangle,$$

(25)

with $A = a f_1(n)$. Even and odd NLCSs for $f$-deformed fields were introduced in [19] and recognized as Shrödinger cat states that are studied in [14]. Nonclassicality features of such superposition states have attracted a great deal of interest in various fields of quantum optics such as quantum computation [40], quantum teleportation [41], precision measurements [42], etc.

4. Generation of a class of SU(2) coherent states

Another interesting property of our proposal is its potential ability to generate a superposition of SU(2) coherent states, which are essentially defined in a finite dimensional space. The modified Pöschl–Teller potential $V(x) = U_0 \tanh^2 (ax)$, as a system that possesses a finite discrete spectrum, has been used in [32], where $U_0$ is the depth of the well and $a$ is its range. The corresponding nonlinearity function reads as

$$f_2(n) = \sqrt{\frac{\hbar a}{2\mu \Omega}} (2s + 1 - n),$$

(26)

where $\mu$ is the reduced mass of the molecule and $s$ is related to the depth, range and mass of the well so that $s(s + 1) = \frac{2\mu U_0}{\hbar^2 a^2}$, where $\mu$ is the reduced mass of the molecule and $s$ is related to the depth and range of the well and the mass molecule according to $s(s + 1) = \frac{2\mu U_0}{\hbar^2 a^2}$.

The subscript 2 in (26) and in what follows refers to the connection with SU(2) coherent states. With the deformed annihilation and creation operators of the modified Pöschl–Teller potential, the operators $K_+^2 = \sqrt{\frac{2\mu a^2}{\hbar^2 U_0}} A_2$, $K_-^2 = \sqrt{\frac{2\mu a^2}{\hbar^2 U_0}} A_2$, and $K_0^2 = n - s$ can be constructed, which satisfy the well-known su(2) Lie algebra, i.e. $[K_+^2, K_-^2] = \pm K_0^2$ and $[K_+^2, K_0^2] = -2K_0^2$. Now it is possible to define the appropriate displacement operator as in (5). The action of such a displacement operator on the vacuum state leads to

$$D_{f_2}(\alpha) | 0 \rangle = \exp (\xi K_+^2 - \xi^* K_-^2) | 0 \rangle,$$

$$\exp (\xi K_+^2) \left( \frac{1}{1 + |\xi|^2} \right)^{-K_0^2} \exp (\xi^* K_-^2) | 0 \rangle$$

$$= \exp (\xi K_0^2) \sum_{n=0}^{\infty} \frac{\Gamma(2s + 1)}{n!\Gamma(2s + 1 - n)} \xi^n | n \rangle,$$

(27)

where we have assumed that $\xi = \sqrt{\frac{\hbar a}{2\mu U_0}}$ and $\xi = \frac{1}{|\xi|} \tanh \xi$.

Also, by redefining $\eta_2(\alpha) = \sqrt{\frac{2\mu a^2}{\hbar^2 U_0}} a$, the resulting state in (27) can be rewritten as

$$| \eta_2(\alpha), f_2 \rangle = \frac{1}{\sqrt{n!}} \sum_{n=0}^{\infty} \eta_2(\alpha)^n f_2(n) | n \rangle.$$

(28)
The obtained equivalent states in (27) and (28) are the well-known SU(2) coherent states and the NLCSs corresponding to finite dimensional space, respectively.

If we substitute the deformed annihilation and creation operators of the modified Pöschl–Teller potential built with \( f_2(n) \) in (26) into the Hamiltonian (7), and assume the atom–field being initially prepared as in (12), then the time evolution of the system arrives at a combination of SU(2) coherent states:

\[
|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\phi/2} e^{-i\phi x} \cos \frac{\theta}{2} |\eta_1(\alpha), f_2\rangle 
+ i e^{-i\phi/2} e^{i\phi x} \sin \frac{\theta}{2} |\eta_2(\alpha), f_2\rangle \right) |e\rangle 
+ \frac{1}{\sqrt{2}} \left( e^{-i\phi/2} e^{-i\phi x} \cos \frac{\theta}{2} |\eta_2(\alpha), f_2\rangle 
- i e^{-i\phi/2} e^{i\phi x} \sin \frac{\theta}{2} |\eta_2(\alpha), f_2\rangle \right) |g\rangle.
\]

(29)

Obviously, as in our discussion at the end of the previous section, by appropriately tuning the phase of the classical field, the individual component, i.e. SU(2) coherent states, can be extracted.

5. Summary and conclusion

In this paper, we proposed a scheme for the generation of a class of NLCSs associated with a particular physical potential, which can also be demonstrated as Gilmore–Perelomov-type coherent states corresponding to the SU(1, 1) group. The proposal employs a system in which a two-level atom simultaneously interacts with a quantized cavity field and a strong external classical field. Also, the interaction with the quantized field is considered to be intensity dependent. The particular \( f \)-deformed annihilation and creation operators (associated with a particular physical potential) which have been used in the paper, together with their commutators, satisfy the su(1, 1) Lie algebra. Therefore, the construction of an \( f \)-deformed (unitary) displacement operator with \( A_{1} \) and \( A_{1}^\dagger \) becomes possible. Under initial conditions which may be prepared (the atom being in a combination of ground and excited states with equal weights and the field being in the vacuum state) the time evolution of the atom–field system generally led to the generation of superpositions of NLCSs (SU(1, 1) coherent states of the Gilmore–Perelomov type). Indeed, after the interaction, if the atom is detected in the excited or ground state, then the state of the cavity field collapses into two distinct superpositions of states. Tuning the accessible parameters, particularly the phase of the classical field, allowed us to generate the individual components of superposed states, i.e. the SU(1, 1) coherent states. On the other hand, if the atom is initially prepared in the excited state and the cavity field is in the vacuum state, by tuning the phase of the classical field, the state of the field arrives at even and odd superpositions of SU(1, 1) coherent states at specific times. As a result, \( f \)-deformed Shrödinger cat states have been produced, whose various applications have been established in the literature. Finally, we showed that, following a similar procedure, but with another physical system (the modified Pöschl–Teller potential), the scheme enabled us to produce a superposition of SU(2) coherent states, or in particular conditions, the individual components, i.e. SU(2) coherent states.

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