Efficient optical trapping with cylindrical vector beams

H. Moradi, V. Shahabadi, E. Madadi, E. Karimi, and F. Hajizadeh

1Department of Physics, Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan 45137-66731, Iran
2Department of Engineering Sciences and Physics, Buin Zahra Technical University, Buin Zahra, Qazvin 3451745346, Iran
3Department of Physics, University of Ottawa, 25 Templeton St., Ottawa, Ontario KIN 6N5, Canada
4Optics Research Center, Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan 45137-66731, Iran
* hajizade@iasbs.ac.ir

Abstract: Radially and azimuthally polarized beams can create needle-like electric and magnetic fields under tight focusing conditions, respectively, and thus have been highly recommended for optical manipulation. There have been reports on the superiority of these beams over the conventional Gaussian beam for providing a larger optical force in single beam optical trap. However, serious discrepancies in their experimental results prevent one from concluding this superiority. Here, we theoretically and experimentally study the impact of different parameters — such as spherical aberration, the numerical aperture of the focusing lens, and the particles’ size — on optical trapping stiffness of radially, azimuthally, and linearly polarized beams. The result of calculations based on generalized Lorenz–Mie theory, which is in good agreement with the experiment, reveals that the studied parameters determine which polarization state has the superiority for optical trapping. Our findings play a crucial role in the development of optical tweezers setups and, in particular, in biophysical applications when laser-induced heating in the optical tweezers applications is the main concern.

© 2019 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

1. Introduction

Optical tweezers, which refer to a single tightly-focused beam for trapping of individual nano- and micro-sized particles in three-dimensions, were introduced by Ashkin and his colleagues in 1986 [1]. By trapping tiny metallic particles down to a size of 10 nanometers, optical tweezers have become a unique method to access the intracellular environment without mechanically disrupting the membrane [2, 3]. Optical tweezers hold promise for applications ranging from nanotechnology to biomolecular actuation force measurements. However, to fully utilize this potential, finding techniques to optimize the system for expanding the working area of optical tweezers, e.g. reducing the heat generation in the trap, is still desired. The most common optical tweezers setups have been formed with a Gaussian mode, i.e. TEM\(_{00}\); however, the other spatial modes of the laser also attracted attention for different applications. For instance, Laguerre-Gaussian modes, which possess a doughnut shape with a null intensity at the centre, confine the metallic nanoparticles in the dark region, and avoid strong absorption and heating [4]. Albeit, it provides less control over the movement of the particles. It has been theoretically and experimentally shown that a radially polarized beam (RPB) — a sub-class of cylindrical vector beams (CVBs), though forms a doughnut shape with a null intensity at the beam centre — would provide smaller focal spot size compared to the conventional Gaussian beam in the tight-focussing regime [5–7]. Because of this exclusive property, RPBs received much attention for different applications, such as optical trapping [8, 9], high-resolution microscopic imaging [10], optical
data storage [11], micromachining [12], just to name a few.

There have been many attempts to explore the advantages and drawbacks of using RPBs as compared to the conventional Gaussian beam in optical trapping. One of the first calculations of the force induced by a RPB shows that the extremely strong axial component of a highly-focused beam only contributes to the gradient force, while the axial Poynting vector near the optical axis is substantially zero [13]. These features of RPBs increase the trapping efficiency significantly, as compared to a linearly polarized beam (LPB), mainly for trapping of metallic nanoparticles for which the scattering and absorption forces are remarkably strong [13]. In contrast to this result, a careful consideration of all the optical forces, including the scattering force that is proportional to the curl of the time-averaged spin density of the light field, does not lead to the superiority of using radial polarization to trap metallic particles [14]. The experimental comparison between the trapping efficiencies of a gold nanoparticle with linearly, radially and azimuthally polarized beams show that the RPB provides the highest trapping stiffness in the transverse plane. The azimuthally polarized beam (APB), on the other hand, gives the lowest stiffness in the transverse plane [8]. The latter lies in the fact that the APB possesses a null of intensity (hollow shape) upon free-space propagation and under tight-focusing condition. When the particle size is bigger than the laser wavelength, a RPB gives a higher axial trapping efficiency in water and air as compared to a LPB [15,16]. However, the transverse trapping efficiency for an APB is larger than that for a RPB [16]. The quality and efficiency of the optical tweezers are determined, to a large extent, by the gradient of the light intensity at the focus. In an ideal conventional system, the sharpness of the focus is diffraction-limited and determined by the numerical aperture (NA) of the focusing lens and the polarization angle of the trapping. For realistic experiments, however, optical aberrations of the microscope lens have a great impact on the gradient force [17–19]. Despite extensive studies, a comprehensive investigation exploring the optimal conditions of trapping with CVBs, and in particular with the radially and azimuthally polarized beams, is still required.

Here, we measure the stiffness of optical traps formed with RPBs and APBs, and compare them to those obtained with LPBs. We also theoretically calculate optical forces with the generalized Lorenz-Mie model. We express the incoming and scattered fields using the T-matrix method, in which the incident and scattered fields are expanded in multipole fields. Furthermore, the influences of several parameters, such as the size of the trapped particle, spherical aberration, and the numerical aperture of the objective lens, are experimentally and theoretically investigated in detail. Our results provide the optimum of the experimental parameters leading to the advantages of using RPBs for optical trapping.

2. Experimental setup

A single particle is optically trapped in the focus formed by a laser beam (Nd:YAG, \( \lambda = 1064 \) nm, Coherent) through a high numerical aperture objective lens (Olympus, UPlan, 100×, NA=1.3, Oil). The sample chamber was mounted on a piezo stage (Physik Instrumente, PI-527.2cl) which, along with a piezo equipped objective holder (Physik Instrumente, P-723.10), provides three-dimensional positioning of the laser focus inside the sample chamber. Figure 1 shows the schematic of the optical tweezers setup. A \( q = 1/2 \)-plate is used to produce CVBs. A \( q \)-plate is a structured liquid crystal device that possesses a desired topological charge \( q \) in its transverse plane [20–22]. When its optical retardation is set to \( \delta = \pi \), \( q \)-plate converts spin-to-orbital angular momentum, and its action in the circular polarization basis, left (\( \hat{e}_L \)) and right (\( \hat{e}_R \)) circular polarizations, is given by

\[
\begin{align*}
\hat{e}_L & \rightarrow \hat{e}_L e^{i2q(\alpha_0 + \alpha)}, \\
\hat{e}_R & \rightarrow \hat{e}_R e^{-2i(\alpha_0 + \alpha)},
\end{align*}
\]
A near-infrared laser ($\lambda = 1064$ nm) is used for trapping. A $q = 1/2$-plate, whose optical retardation is set to $\pi$ by means of a signal generator, is used to generate cylindrical vector beams. A quadrant photodiode (QPD) measures the position of a trapped bead by back-focal-plane detection, which is amplified before recording it with a computer.

Figure legends: L: Lens, M: Mirror, HWP: half wave plate, DM: Dichroic mirror, Obj: objective lens, QPD: quadrant photodiode. The dotted inset shows a zooming in view of the focus.

Here $\alpha_0$ is a real constant and $\varphi$ is the azimuth angle in the cylindrical coordinates. Thus, a $q = 1/2$-plate with $\alpha_0 = 0$, converts linearly polarized input beams $(\hat{e}_L \pm \hat{e}_R)/\sqrt{2}$ into CVBs $(\hat{e}_R e^{i\varphi} \pm \hat{e}_L e^{-i\varphi})/\sqrt{2}$. To tune the $q$-plate, we followed the method presented in [22], for which we tuned the optical retardation ($\delta = \pi$) of the $q$-plate with an AC voltage produced by a signal generator, instead of with temperature [23]. The measurements for LPBs were also performed with a $q$-plate, as well, but it was detuned ($\delta = 0$ or $2\pi$) to keep the optical retardation to be integer multiples of $2\pi$. To prevent changes in the polarization state of the laser beam by the dichroic mirror (DM1) [24], the $q$-plate is mounted in the light path before the objective lens. The lateral intensity distribution of the generated beams after the $q$-plate are shown in Fig. 2, in which 2(a) and 2(b) are the Gaussian beam and CVB, respectively. The polarization states of the generated RPB and APB are tested using a rotated linear polarizer, Figs. 2(c) and 2(d), respectively, show the intensity of the RPB and APB after a horizontal polarizer. To be sure that the generated beams are in fact RPB and APB, the bottom panel in Fig. 2 shows the intensity distribution of the CVBs after passing through the various orientations of the polarizer.

The chamber was made of a coverslip, as bottom surface, a microscope slide, as top surface, and double-sided sticky tape with a thickness of $\sim 100 \mu$m as a spacer. Polystyrene beads, with the same refractive index, in different sizes of 0.50, 0.80, 1.09, 1.26, 1.48, 1.65 and 2.10 $\mu$m in diameter, purchased from Sigma Aldrich, were used for the measurements. Once the particle was trapped at the depth of 3 $\mu$m from the inner surface of the coverslip, the positional signal from the photodiode was recorded for $\sim 3$ seconds for each measurement. Data recording was performed with a custom-made LabVIEW programs, and then a MATLAB program was used to fit a Lorentzian function to the power spectrum of the recorded data, leading to the trapping corner frequency [25]. Trap stiffness can be written as $K = \frac{12\pi \eta r f_c}{\zeta}$, with $\eta$, $r$, and $f_c$ being viscosity, particle radius, and corner frequency, respectively. It is worth mentioning that, in the case of the azimuthal polarization, the focal spot has an annular intensity distribution (hollow shape) and the particles with sizes smaller than the dark spot can be trapped in the focal ring [26]. However, we
noted that all sizes of the particles in this study trapped in the center of the transverse spot of the laser beam, which means the particles are bigger than the dark spot of the focal point. We also tried to investigate the influence of changing the NA on the stiffness of the trap. For this aim, we used a variable NA objective (Zeiss, PlanApo, 63×, NA=0.7-1.4, Oil) for trapping the particles. The variable NA objective has an iris able to adjust the numerical aperture. It means that the lower numerical aperture provides less laser power; therefore, the measured stiffnesses are normalized to the laser power at the sample. Oil objectives, with higher NA objectives than the water or air objectives, are optically corrected to provide a tighter focus very close to the coverslip. When a particle is trapped at a distance from the coverslip, the spherical aberration, caused by a refractive index mismatch between the coverslip and water, makes a weaker trapping efficiency. Trapping particles at a distance from the coverslip is the most common application of the optical trap in micro-fluidics and experimental biology. Therefore, it is important to choose the right depth or to compensate the aberration for the desired depth by a careful adjustment of the immersion oil index when manipulation is done far away from the coverslip [19].

3. Theoretical approach

Radially $E_r(r)$ and azimuthally $E_\phi(r)$ polarized beams, as axially symmetric beams, are the solutions to the vector Helmholtz equation in the paraxial limit. These beams are coherent superpositions of the lowest order orthogonal modes of Hermite-Gaussian beams possessing orthogonal linear polarizations [27]:

$$E_r(r) = HG_{10}(r) \hat{e}_x + HG_{01}(r) \hat{e}_y,$$

$$E_\phi(r) = HG_{01}(r) \hat{e}_x - HG_{10}(r) \hat{e}_y,$$

where $\hat{e}_x, \hat{e}_y$ are unit vectors along $x$- and $y$-axis, and $HG_{mm}(r)$ are the Hermite-Gaussian modes [28], respectively. In an optical tweezers setup, incoming laser light beam focuses to a diffraction limited spot utilizing high-numerical aperture objective lens. By applying the vectorial Debye diffraction integral [29], a three-dimensional distribution of an electric field in the focus
of an objective lens using Cartesian coordinates can be expressed as:

$$\mathbf{E}_v(r) = \left( \frac{im_1 k_0 e^{im \kappa_0 f}}{2\pi} \right) \int_0^{2\pi} \int_0^\alpha d\phi d\theta_1 \sin \theta_1 \mathbf{E}_{\text{sample}}^{(v)}(\theta_1, \phi) e^{im \kappa_0 r} e^{i \phi},$$

(3)

where subscript $v=1, 2$ stands for radial (azimuthal) polarization, and $k_0, n_1, n_2,$ and $\alpha$ are respectively the vacuum wavenumber, refractive index of the objective, refractive index of the sample medium, and the maximum converging angle viewed from the focus. Furthermore, $f$ is the focal length, $\gamma = f/w_0$ with $w_0$ being the beam waist in the pupil, and

$$\mathbf{E}_{\text{sample}}^{(v)} = \frac{2E_0 \gamma}{\sqrt{\mu_1}} \sin \theta_1 \sqrt{\cos \theta_1} e^{-\gamma^2 \sin^2 \theta_1} \tau^{(v)} \hat{c}_v$$

is the electric field in the sample medium. $\tau^{(1)}$ and $\tau^{(2)}$ are Fresnel’s transmission coefficients for $p$ and $s$ polarizations, $\hat{c}_1$ and $\hat{c}_2$ correspond to the polar and azimuthal directions, and $\psi = kd(n_1 \cos \theta_1 - n_2 \cos \theta_2)$ is the spherical aberration, originating from the optical pathway refractive index mismatch. Here, $d, \theta_1, \theta_2$ respectively refer to the distance between laser focus and inner surface of the coverslip, the incident and refracted angles on the separating surface of the media, which are connected by Snell’s law (see Fig. 1 inset).

Figure 3 shows the theoretically simulated electric energy density distribution in the axial direction for LPB, RPB, and APB, in the presence and the absence of the spherical aberration. For this simulation, $w_0, n_1, n_2,$ and $\alpha$ are assumed to be 4 mm, 1.52, 1.33, and 59°, respectively. The dark region radius and $w_0$ in the RPB and APB used in the theory simulation have been chosen to be equal to the measured values in the experimental setup. As Fig. 3 shows for optical trapping in a deeper focus, the spherical aberration dramatically increases, causing a weaker trapping efficiency. It could be seen that the intensity distribution of focus formed by RPB and APB as well as LPB are narrower in the absence of the aberration. Moving away from the coverslip means increasing the spherical aberration, makes the Full Width at Half Maximum (FWHM) of each three polarization states, i.e. LPB, RPB, and APB, to be wider. For instance, lateral FWHM for the LPB, RPB, and APB, from 1.10, 1, and 1.05 μm, increases to 1.77 μm, 1.80 μm, 1.77 μm, respectively, as the depth increases from $d=0$ to 10 μm.

Generalized Lorenz-Mie theory (GLMT) is a powerful method to calculate the scattered fields from a trapped spherical particle. In the GLMT, incident and scattered fields are expanded in vector spherical harmonics, since they form a complete basis [30]:

$$\mathbf{E}_i = \sum_{p \mu |lm} \Psi^{(p)}_{lm}(\mu, \phi) \mathbf{Y}^{(p)}_{lm}, \quad \mathbf{E}_s = \sum_{p \mu |lm} \mathcal{A}^{(p)}_{lm}(\mu, \phi) \mathbf{H}^{(p)}_{lm},$$

(4)

where $p = 1$ and 2 are respectively the magnetic and electric multipoles, $\Psi^{(p)}_{lm}$ are expansion coefficients of the incoming fields which are extracted from Eq. (3), and $\mathcal{A}^{(p)}_{lm}$ are expansion coefficients of the scattered fields that can be calculated by applying customary boundary conditions on the surface of the spherical particle [31]. $\mathbf{J}^{(p)}_{lm}$ ($\mathbf{H}^{(p)}_{lm}$) are the multipole fields of the incident waves (scattered waves) [32]. To calculate the force exerted on a particle, $\mathbf{F}$, it is convenient to integrate numerically over Maxwell’s stress tensor $\mathbf{T}$ [33]:

$$\mathbf{F} = \int_S \hat{\mathbf{r}} \cdot \mathbf{T} dA$$

$$\langle \mathbf{T} \rangle = \frac{\mu_0}{2} \left[ n_1^2 \mathbf{E} \otimes \mathbf{E}^* + c^2 \mathbf{B} \otimes \mathbf{B}^* - \frac{1}{2} \left( n_1^2 |\mathbf{E}|^2 + c^2 |\mathbf{B}|^2 \right) \right].$$

Here, $\otimes$, $c$, and $\mathbf{I}$ are dyadic multiplication, $*$ stands for the complex conjugate, the speed of light in vacuum and unit dyadic, respectively. The total magnetic and electric fields are the sum of the incident and scattered fields, i.e. $\mathbf{B} = \mathbf{B}_i + \mathbf{B}_s$ and $\mathbf{E} = \mathbf{E}_i + \mathbf{E}_s$. 
4. Results and discussion

The experimental results and theoretical simulations of optical trapping stiffness formed by the LPB, RPB, and APB are shown in Fig. 4 as a function of the particle size. All the measurements are performed at the same distance, ~3 \( \mu m \), from the coverslip which has the highest stiffness of the trap (minimal aberration). Note that the lateral stiffness for the LPB exhibits an asymmetry due to the polarization state of the beam; therefore, all the lateral stiffness results reported here are the average of the stiffness for the \( x \) and \( y \) directions [34].

Figure 4 illustrates that: 1) Both the theory and the experiment exhibit a similar trend of rise and fall in the graphs as the particle size varies, although, there are slight differences in the values. There are a number of reasons that may create the differences. The RPB and APB generated using a q-plate in the lab are not ideally as the same as one considered by theory. For instance, in the Fig. 2(b), the intensity profile of CVB shows a diffraction pattern around the dark spot which causes a clear difference in the intensity profile from the theory. Another reason would be the Brownian motion of the particle, which is not considered in theory. 2) The particle size determines which polarization state has a higher efficiency. The LPB provides a stronger trap for particles with \( 0.50 \ \mu m \) and \( 0.80 \ \mu m \) diameters for both axial and lateral directions. However, the trapping stiffness of the RPB for bigger particles dominates than that of the LPB. Choosing the RPB to trap a particle with \( 2.10 \ \mu m \) diameter, as opposed to the LPB, enhances the axial efficiency by a factor of ~ 2.63 (2.44) based on experimental (theoretical) results. Furthermore,
Fig. 4. Experimental results (left panel) and theoretical simulations (right panel) of trap stiffness normalized to the laser power at the sample along the axial ((a) and (b)) and lateral ((c) and (d)) directions as a function of the particle diameter, based on the company specifications. The blue triangles (solid lines), red squares (dashed lines), and black circles (dash-dotted lines) indicate the trapping stiffness in experiment (theory) for the LPB, RPB, and APB, respectively. Error bars in the experimental panel show the standard deviations of the measured values (at least 9 measurements; 3 particles and 3 measurements for each).

the advantage of the RPB to the LPB for optical trapping of a particle with 2.10 µm diameter in comparison with a particle with 0.5 µm is a factor of 4.6 (5.9) for axial (lateral), based on experimental results. In the other polarization angles of CVBs (hybrid modes), as reported in [17], the optical trapping stiffness reaches to its extremum values when the polarization angle corresponds to the polarization state of radial or azimuthal polarizations. Accordingly, we think using hybrid modes is not helpful to reach maximum trapping efficiency and have not been studied here. 3) In comparison to the trend of the stiffness for the LPB, the trend for the stiffness values for the RPB and APB are similar as the particle size varies. However, the higher stiffness of the RPB in the axial direction (Fig. 4(a) and Fig. 4(b)) compared to the APB exhibits a stronger axial component of the electric field under tight focussing conditions [6]. 4) The intersection of the graphs for the RPB and LPB almost occurs at particle sizes equal to the laser wavelength (∼ 1 µm) where they change the trapping efficiency compared to each other. As we observed in all the measurements, particles always trapped in the center of the focal point, even for the APB which is doughnut shaped at the focus (Fig. 3). Although, trapping particles inside the annular intensity of the APB has been reported in [26], we have not observed any trapping in the focal ring even for the smallest particle with a diameter of 0.50 µm. It seems that the dark spot of the APB focus is smaller than the smallest particle (0.50 µm) studied in this work.

Figure 5 shows a size-dependent axial and lateral stiffness asymmetry measured [17] for the LPB, RPB, and APB. Because of the dark region at the focus for the APB, the optical trap is weak in the axial direction and causes a considerable axial asymmetry for the APB as shown in Fig. 5(a). However, it is not predicted by the theory in Fig. 5(b). The experimental lateral
symmetry for the APB and RPB in Fig. 5(c) shows a symmetric experimental light distribution at
the focus for these two beams, which is zero in the theoretical result Fig. 5(d). A slight deviation
from zero for the LPB reveals a size-dependent polarization-induced asymmetry of the laser
beam [34].

The effect of the numerical aperture on the trapping efficiency formed by a LPB, RPB, APB,
as well as a circularly polarized beam (CPB) are experimentally and theoretically investigated for
the 0.50 µm (1.26 µm) diameter particles smaller (bigger) than the laser wavelength (Fig. 6).
These results, shown in Fig. 6, clarifies that, regardless of the numerical aperture (NA), the
conventional Gaussian beam has superiority to trap particles smaller than 1 µm. As could be seen
in Figs. 6(a)-6(d), the superiority of the LPB to RPB for trapping a particle with the diameter of
0.50 µm does not change when the NA varies. However, the efficiency of the trap formed by

![Fig. 5. Experimental results (left panel) and theoretical simulations (right panel) of axial ((a)
and (b)) and lateral ((c) and (d)) asymmetry of the trap as a function of the particle diameter.]

![Fig. 6. Experimental results and theoretical simulations of the trapping stiffness normalized
to the laser power at the sample along the axial (top panel) and lateral (bottom panel)
directions as a function of the objective numerical aperture for 0.5 µm ((a)-(d)) and 1.26 µm
((e)-(h)) particles.]

Vol. 27, No. 5 | 4 Mar 2019 | OPTICS EXPRESS 7273
Fig. 7. The effect of spherical aberration. Experimental (left panel) and Theoretical (right panel) results of the trap stiffness normalized to the laser power at the sample for the LPB (i), RPB (ii), and APB (iii) as a function of the trapping depth. The error bars represents the standard deviation of at least 9 measurements.

the RPB in comparison to a Gaussian beam (both LPB and CPB) for the particle with 1.26 \( \mu m \) diameter depends on the NA and dominates only for NAs bigger than 1.1 (Figs. 6(e) and 6(f)).

Finally, in order to study the spherical aberration induced by refractive index mismatch between oil and water, the particle was trapped at the depth of 3 \( \mu m \) from the inner surface of the coverslip, and then it was pushed away toward the top surface (Fig. 1(inset)), while the positional signal from photodiode was recorded every one micrometer. Figure 7 demonstrates the trapping stiffness for laser tweezers made by a LPB, RPB, and APB as a function of trapping depth. The figure shows that all the polarization states provide a maximal stiffness at a certain depth of \( \sim 3 \mu m \) [19] and drops for larger depths. However, the rate of the reduction depends on the particle size and polarization state. It is worth mentioning that the APB, in comparison to two other polarization distributions, can trap a particle in a narrower depth band using oil immersion objectives; however, the depth band is broader for the LPB and RPB. As it can be seen from the graphs, the particle with the diameter of 1.65 \( \mu m \) was stably trapped in three-dimensions up to the depth of 15 \( \mu m \) by the RPB and LPB. Therefore, to use the ultimate efficiency of the trap,
it is important to consider spherical aberrations in order to choose the right depth/polarization distribution for the manipulation experiments.

5. Conclusion

In summary, we have used CVBs, and in particular RPB and APB, to study the optical trapping stiffness for several particle sizes in the range of \( \sim 0.5 - 2 \mu m \), which are the sizes commonly used for optical manipulations, e.g. living cell studies. The results reveal that the implemented parameters, such as particle size, numerical aperture and the depth of trapping, specify which polarization state has more efficiency for optical trapping. RPBs are superior for trapping particles bigger than the wavelength and with a numerical aperture higher than 1.1. Furthermore, APBs are not a good choice for manipulations needed to be done far away from the coverslip. These results could be easily considered in optical tweezers setup to reduce the laser based destruction, such as heating, by changing the polarization distribution for different particle sizes. Our study shows that choosing the right particle size or polarization distribution enhances the trapping efficiency by a factor of \( \sim 2.5 \).

Funding

Canada Research Chairs; Canada Foundation for Innovation (CFI); Natural Sciences and Engineering Research Council of Canada (NSERC).

Acknowledgments

The authors would like to thank H. Larocque and M. Mousavi for their helpful discussions and assistance.

References

31. C. F. Bohren and D. Huffman, Absorption and Scattering of Light by Small Particles (Wiley, 1983).